NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science Honours in Applied Mathematics	
QUALIFICATION CODE: 08BSHM	**LEVEL:** 8
COURSE CODE: ANA801S	**COURSE NAME:** APPLIED NUMERICAL ANALYSIS
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 120 (to be converted to 100%)

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF S. A. REJU
MODERATOR:	PROF S. MOTSA

INSTRUCTIONS

1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencils.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [30 MARKS]

(a) Consider following integral:

$$A = \int_c^d f(x)dx \tag{1.1}$$

State the general Composite rule and hence the Composite Trapezoidal rule and the Romberg's Method for solving (1.1); and hence using the unit interval [0, 1] for the integral

$$T(n) = \int_a^b f(x)dx$$

and step size

$$h = \frac{(b-a)}{n}$$

obtain the term for the recursive expression $T(2^n) = T(8)$ and the expression for $R(n, 0)$ denoting the Trapezoidal estimate with 2^n. **[20]**

(b) By just stating the Richardson's Extrapolation $R(n, m)$ employed in the Romberg's Table, show that

$$R(1,0) = \frac{1}{2}R(0,0) + \frac{1}{2}(b-a)f\left(\frac{a+b}{2}\right)$$

[10]

QUESTION 2 [35 MARKS]

(a) Derive the Forward Euler's Method, using any appropriate diagram for substantiating your discussion. **[13]**

(b) Consider the following IVP:

$$\frac{dy(t)}{dt} + ay(t) = r, \quad y(0) = y_0 \tag{2.1}$$

(i) State the Euler's method that approximates the derivative in the above equation and hence state the resulting difference equations with three stepwise increments of t by h from $t = 0$.

(ii) Taking $a = 1 = r$ and $y_0 = 0$, obtain the numerical solutions of (2.1) for $t = 0.25, \ldots, 1.5$ when $h = 0.25$ and $h = 0.5$ (correct to 4 decimal places)

[22]

QUESTION 3 [30 MARKS]

(a) State the pseudo code for the Conjugate Gradient Method (CGM) for solving the nxn system of linear equations:

$$Ax = b$$

where A is a symmetric and positive definite matrix. **[10]**

(b) Consider the following system of linear equations:

$$\begin{bmatrix} 5 & -2 & 0 \\ -2 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ -10 \end{bmatrix}$$

Solve the above system using the Conjugate Gradient Method using the initial vector:

$$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[20]

QUESTION 4 [25 MARKS]

(a) Discuss the contrast between a quadrature rule and the adaptive rule. **[3]**

(b) Consider the integral **[27]**

$$\int_a^b f(x)dx = \int_1^3 e^{2x} sin(3x)dx$$

Using the Adaptive Simpson's Method and an error $\epsilon = 0.2$, obtain the approximate value of the above integral (for computational ease, using where appropriate the following as done in class):

$$\frac{1}{10}\left|S(a,b) - S(a,\frac{a+b}{2}) - S(\frac{a+b}{2},b)\right|$$

where

$$\int_a^b f(x)dx = (S(a,b) - \frac{h^5}{90} f^{(4)}(\xi), \ \ \xi \epsilon (a,b)$$

END OF QUESTION PAPER TOTAL MARKS = 120